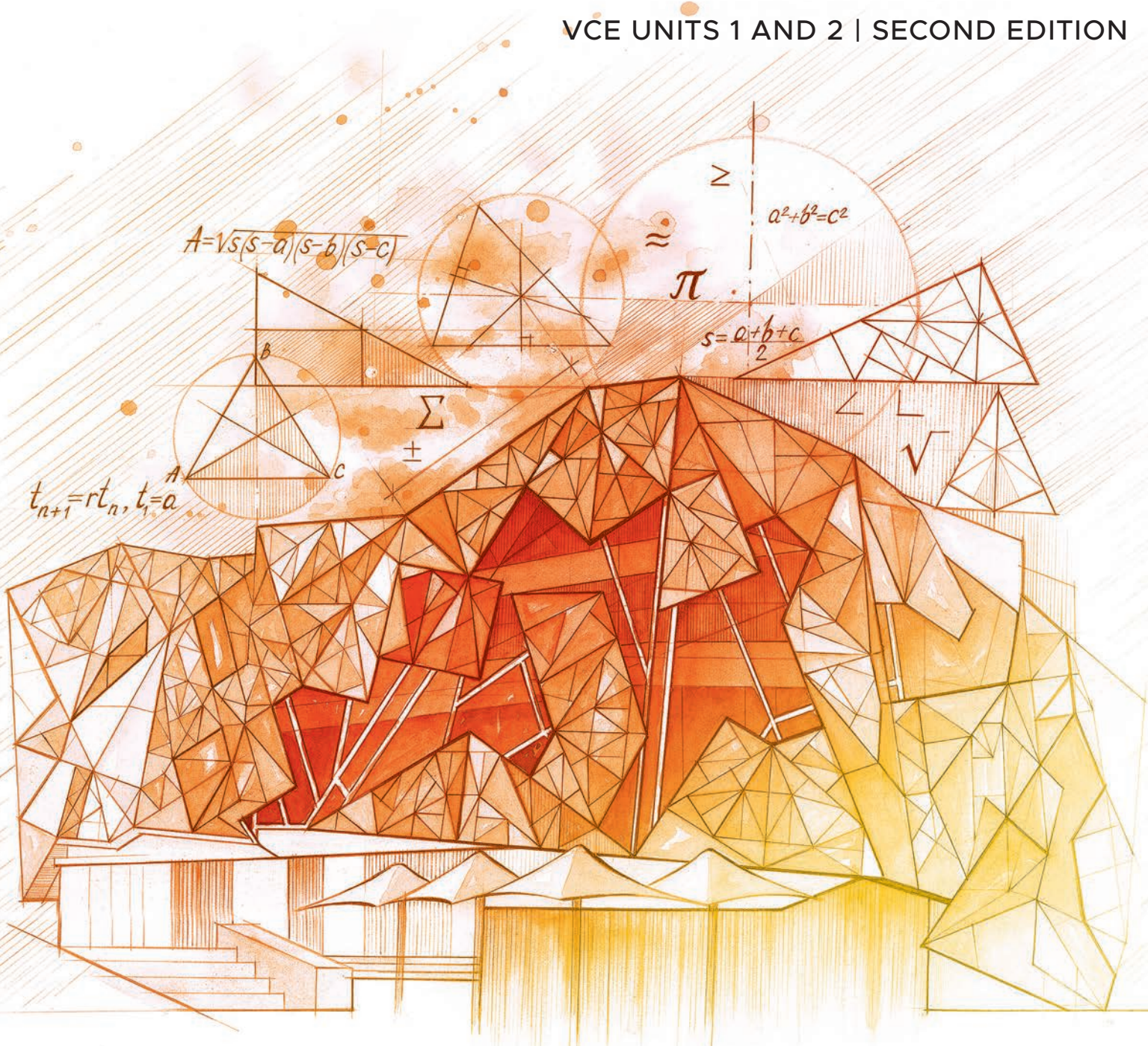


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# JACARANDA MATHS QUEST FOUNDATION MATHEMATICS 11

VCE UNITS 1 AND 2 | SECOND EDITION



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## UNITS 1&2 MATHS QUEST 11 FOUNDATION MATHEMATICS

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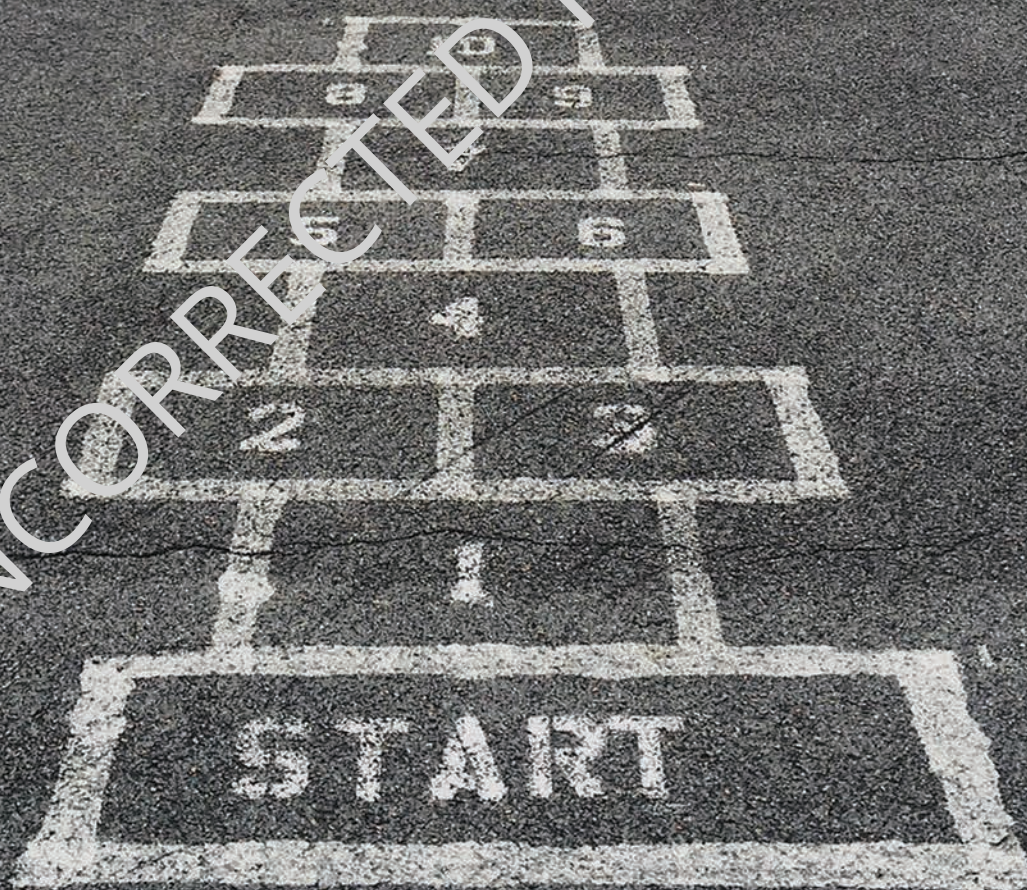


# 1 Integers

## LEARNING SEQUENCE

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Fully worked solutions for this topic are available online.





# 1.1 Overview

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## 1.1.1 Introduction

An integer is a whole number (e.g. 100, 1, 0,  $-1$  and  $-100$ ), with no fractions or parts of a whole included. They can be positive (above zero), negative (below zero) or equal to zero.

Being able to add, subtract, multiply and divide integers is important in many parts of everyday life. It helps with budgeting and knowing what you can and can't afford to buy — or how much of something to buy. It also helps you to keep safe; for example, being able to read the integers on the speedometer helps you stay within the speed limit. It is also useful for understanding temperatures, weights and measures when you're cooking or you need to store food safely.

In finance, positive numbers are used to represent the amount of money in someone's bank account, while negative numbers are used to represent how much money someone owes (for example, how much they have to pay back after borrowing money or taking out a loan from a bank).



### KEY CONCEPTS

This topic covers the following key concepts from the VCE Mathematics Study Design:

- application of integers, fractions and decimals, their properties and related operations
- use of ratios, proportions, percentages and rates to solve problems
- estimation, approximation and reasonableness of calculations and results.

**Note:** Concepts shown in grey are covered in other topics.

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# 1.2 Adding and subtracting integers

## LEARNING INTENTION

At the end of this subtopic you should be able to:

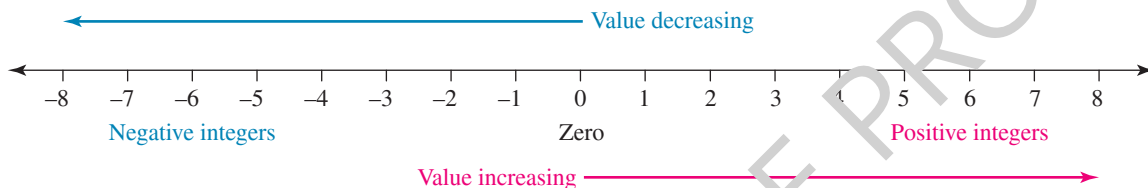
- identify positive and negative integers
- add and subtract integers.



eles-0040

## 1.2.1 Addition and subtraction of integers using a number line

An integer is a whole number. It can be positive (2, 4, 89, 1035) or negative (−2, −4, −89, −1035). Zero is also an integer because it is a whole number.



The positive numbers do not usually have a symbol to show that they are positive (+2 is the same as 2). However, the negative numbers must include the negative sign to show that they are negative (−2, −27).

A whole number is a number that does not include any fractions or parts of a number. For example, the following are *not* whole numbers or integers:

- Fractions ( $\frac{1}{2}$ ,  $-12\frac{3}{4}$ ,  $5\frac{1}{2}$ ) because they are or include parts of a whole number
- Numbers that continue after a decimal point (0.5, −12.75, 5.5)

Number lines can be used to remember the rules for adding and subtracting integers.

### Addition and subtraction using a number line

**Start at the first number.**

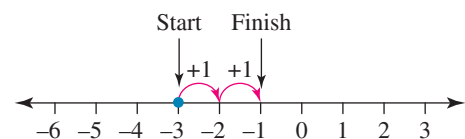
**To add a:**

- **positive integer, move to the right**
- **negative integer, move to the left.**

**To subtract a:**

- **positive integer, move to the left**
- **negative integer, move to the right.**

To show  $-3 + (+2)$  on a number line, start at the first number (−3) and then place a pointer on the number line at −3. To add the positive integer (+2), move 2 places to the right.



Therefore, using the number line, we can see that  $-3 + (+2) = -1$ .

### WORKED EXAMPLE 1 Adding and subtracting integers using a number line

Use a number line showing intervals from −6 to 3 to calculate each of the following.

a.  $2 + (+1)$

b.  $3 + (-2)$

c.  $-4 - (+2)$

d.  $-3 - (-5)$



**THINK**

a. 1. Start at 2 and move 1 unit to the right, as this is the addition of a positive number.

2. Write the answer.

b. 1. Start at 3 and move 2 units to the left, as this is the addition of a negative number.

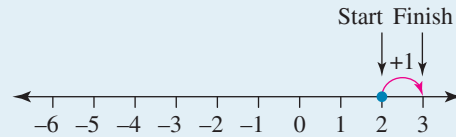
2. Write the answer.

c. 1. Start at  $-4$  and move 2 units to the left, as this is the subtraction of a positive number.

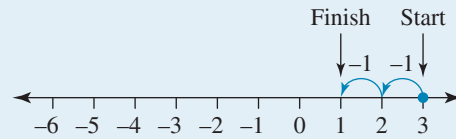
2. Write the answer.

d. 1. Start at  $-3$  and move 5 units to the right, as this is the subtraction of a negative number.

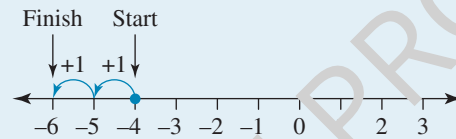
2. Write the answer.

**WRITE**

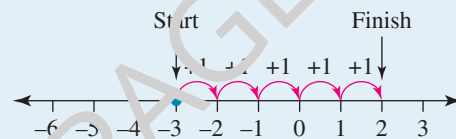
$$2 + (+1) = 3$$



$$3 + (-2) = 1$$



$$-4 - (+2) = -6$$



$$-3 - (-5) = 2$$

## 1.2.2 Addition and subtraction of integers using symbols

eles-xxxx

Another method of remembering the rules for adding and subtracting integers is to imagine positive and negative symbols or signs representing each of the numbers in an equation.

When calculating:

- positive symbols collect together; 2 is written as:

⊕ ⊕

- negative symbols collect together;  $-2$  is written as:

⊖ ⊖

- positive and negative symbols cancel each other out:  $-1 + 1 = 0$

⊖ ⊕

### WORKED EXAMPLE 2 Adding and subtracting integers using symbols

Calculate the value of each of the following.

a.  $2 + 2$

b.  $-2 + 2$

c.  $-2 - (3)$

d.  $-4 - (-2)$

**THINK**

- a. 1. Set the equation up using symbols.  
2. Add all the positives.  
3. Write the answer.
- b. 1. Set the equation up using symbols.  
2. Cancel any pairs of positives and negatives.  
3. Count how many symbols are left.  
4. Write the answer.
- c. 1. Set the equation up using symbols.  
2. When subtracting positive integers, change the symbols' sign and then add the symbols.  
3. Count all negatives.  
4. Write the answer.
- d. 1. Set the equation up using symbols.  
2. When subtracting negative integers, change the symbol's sign and then add the symbols.  
3. Cancel any pairs of positive and negative symbols.  
4. Count how many positives or negatives remain.  
5. Write the answer.

**WRITE**

$$2 + 2 = \text{+ + + +}$$

$$4 \text{ positives} = +4$$

$$2 + 2 = 4$$

$$-2 + 2 = \text{- - + +}$$

$$\cancel{\text{-}} \cancel{\text{-}} + \cancel{\text{+}} \cancel{\text{+}}$$

Zero symbols remain.

$$-2 + 2 = 0$$

$$-2 - (3) = \text{- - - + + +}$$

$$\text{- - + - -}$$

$$5 \text{ negatives} = -5$$

$$-2 - (3) = -5$$

$$-4 - (-2) = \text{- - - - - - -}$$

$$\text{- - - - + +}$$

$$\cancel{\text{-}} \cancel{\text{-}} - \text{- -} + \cancel{\text{+}} \cancel{\text{+}}$$

Two negatives remain.

$$-4 - (-2) = -2$$

 1.2.3 Addition and subtraction of integers by applying rules

eles-xxxx

You can also follow the rules shown in this table to add and subtract positive and negative numbers.

### Rules for addition and subtraction of integers

Rule		Example
Adding a positive integer is the same as adding.	$++ = +$	$+2 + (+5) = +7$
Adding a negative integer is the same as subtracting.	$+ - = -$	$+2 + (-5) = -3$
Subtracting a positive integer is the same as subtracting.	$- + = -$	$+2 - (+5) = -3$
Subtracting a negative integer is the same as adding.	$- - = +$	$-2 - (-5) = 3$

### WORKED EXAMPLE 3 Adding and subtracting integers by applying rules

Calculate the value of each of the following.

a.  $2 + 2$

b.  $2 + (-2)$

c.  $2 - (+3)$

d.  $-4 - (-2)$

**THINK**

- a. 1. Adding a positive integer is the same as adding.  
2. Apply the rule  $++ = +$  and calculate the value.

**WRITE**




$$2 + 2$$

$$2 + 2 = 4$$




- b.** 1. Adding a negative integer is the same as subtracting.  $2 + (-2)$   
 2. Apply the rule  $+ - = -$  and calculate the value.  $2 + (-2) = 2 - 2$   
 3. Write the answer.  $2 - 2 = 0$
- c.** 1. Subtracting a positive integer is the same as subtracting.  $2 - (+3)$   
 2. Apply the rule  $+ - = -$  and calculate the value.  $2 - (+3) = 2 - 3$   
 3. Write the answer.  $2 - 3 = -1$
- d.** 1. Subtracting a negative integer is the same as adding.  $-4 - (-2)$   
 2. Apply the rule  $- - = +$  and calculate the value.  $-4 - (-2) = -4 + 2$   
 3. Write the answer.  $-4 + 2 = -2$

## on Resources


-  **Digital documents** SkillsHEET Integers on the number line (doc-6387)  
 SkillsHEET Adding and subtracting integers (doc-6388)  
 SkillsHEET Arranging numbers and order (doc-6389)
-  **Video eLesson** Integers on the number line (eles-0040)
-  **Interactivities** Direct number target (int-0074)  
 Addition of positive integers (int-3922)  
 Subtraction of positive integers (int-3924)

## 1.2 Exercise

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1. Select which of the following numbers are integers.

$$5, -2, \frac{3}{4}, 212, 12.3, -2.5, -33, -2\frac{1}{2}$$

2. **WE1** Calculate each of the following using the number line method.

a.  $-5 + 2$

b.  $-6 + 1$

c.  $5 + -3$

d.  $-8 + -3$

e.  $21 + +9$

f.  $18 + -5$

3. **WE2** Calculate each of the following using the cancelling signs method.

a.  $5 + +1$

b.  $3 + -2$

c.  $4 - +5$

d.  $12 + -2$

e.  $3 - -2$

f.  $-1 - -3$

4. Calculate the value of each of the following.

a.  $-7 + -3$

b.  $8 - -7$

c.  $-23 + +15$

d.  $-18 - -17$

e.  $26 - -13$

f.  $-72 - -26$

5. Calculate the value of each of the following using the method of your choice.

a.  $-3 + -2$

b.  $-7 + -3$

c.  $8 + -2$

d.  $15 - +6$

e.  $-15 - -4$

f.  $-23 + -14$

6. Calculate the value of each of the following using the method of your choice.

a.  $-37 + -12$

b.  $42 - +7$

c.  $-14 - +18$

d.  $-27 - -15$

e.  $37 - +12$

f.  $135 - -37$

7. Calculate the value of each of the following using the method of your choice.

a.  $12 + -4 + +6$

b.  $28 + -7 - -10$

c.  $-15 + +5 - +8$

d.  $28 - +15 - +4$

e.  $18 - -12 + -5$

f.  $-42 - -21 - -21$

8. Write out these equations, filling in the missing numbers.

a.  $-5 + \square = 2$

b.  $12 - \square = 8$

c.  $-8 - \square = -20$

d.  $30 + \square = 25$

e.  $-2 - \square - -8 = 2$

f.  $\square - 3 + 5 = 7$

9. State whether the following expressions are True or False.

a.  $-5 + -10 = -15$

b.  $-7 - -6 = -13$

c.  $-4 - 3 = 7$

d.  $8 - -6 + -10 = 4$

e.  $50 + 12 - -12 = 50$

f.  $-23 + 10 + 6 = 7$

10. Evaluate and compare the following pairs of expressions.

a.  $-7 + 3$  and  $3 - 7$

b.  $-8 + 6$  and  $6 - 8$

c.  $-15 - 5$  and  $5 - 15$

11. Evaluate and compare the following pairs of expressions.

a.  $-3 + -4$  and  $-(3 + 4)$

b.  $-8 + -3$  and  $-(8 + 3)$

c.  $-10 - +3$  and  $-(10 + 3)$

12. Layla is standing in a park. She runs 20 m to the right, then 5 m to the left, before running another 45 m to the right. Use a number line to show where Layla finishes compared to where she started.

13. Bryce decides to cook meat pies for lunch. He takes the pie out of the freezer that is set at  $-18^\circ\text{C}$  to defrost on the bench where the room temperature is  $21^\circ\text{C}$ . He sets the oven to  $180^\circ\text{C}$  to cook the pies.

a. State the difference between the temperature of the freezer and the room temperature.

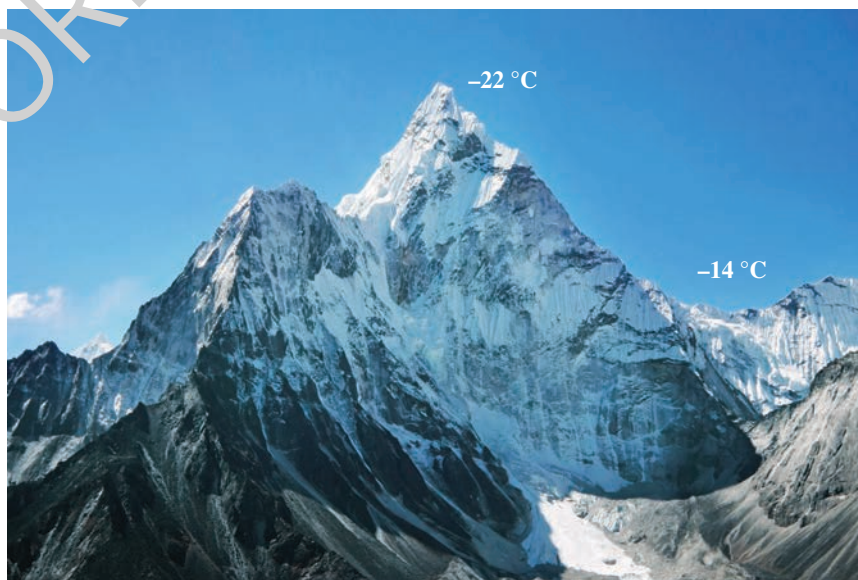
(Hint: difference = largest number  $-$  smallest number)

b. Calculate the difference between the room temperature and the temperature of the oven.

c. Calculate the difference of the freezer temperature and the temperature of the oven.



14. Calculate the difference between the two temperatures on Mount Everest shown in the image.





## 1.3 Multiplying integers

### LEARNING INTENTION

At the end of this subtopic you should be able to:

- multiply integers
- evaluate powers, squares and square roots of integers.

### 1.3.1 Multiplication of integers

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When multiplying integers, the following rules apply.

#### Rules for the multiplication of integers

- When multiplying two integers with the same sign, the answer is positive

$$\begin{aligned} + \times + &= + \\ - \times - &= + \end{aligned}$$

- When multiplying two integers with different signs, the answer is negative.

$$\begin{aligned} + \times - &= - \\ - \times + &= - \end{aligned}$$

#### WORKED EXAMPLE 4 Multiplying integers by applying rules

Evaluate each of the following.

a.  $-4 \times +3$

b.  $-7 \times -6$

**THINK**

- a. The two numbers have different signs, so the answer is negative.
- b. The two numbers have the same sign, so the answer is positive.

**WRITE**

a.  $-4 \times +3$   
 $= -12$

b.  $-7 \times -6$   
 $= 42$  (or  $+42$ )

### 1.3.2 Powers, squares and square roots of integers

eles-xxxx

#### Powers

A power of a number is the number multiplied by itself multiple times. For example,  $8^2 = 8 \times 8$  and  $(-6)^3 = -6 \times -6 \times -6$ .

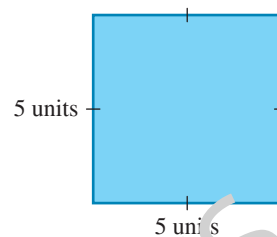
A power is represented with a base number and an exponent. When a negative number is raised to a power, the sign of the answer will be:

- positive, if the power is even;  
e.g.  $(-5)^2 = -5 \times -5 = +25$
- negative, if the power is odd;  
e.g.  $(-5)^3 = -5 \times -5 \times -5 = +25 \times -5 = -125$

## Squares

A square number is any whole number multiplied by itself. When written in index form, it will have a power of 2.

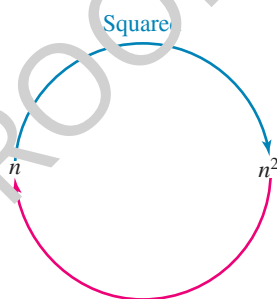
A square number can be illustrated by looking at the area of a square with a whole number as its side length. Consider the image shown. We can say that  $5^2$  or 25 is a square number, since  $5^2$  or  $25 = 5 \times 5$ .



## Square roots

The square root of a number is a positive value that, when multiplied by itself, gives the original number. The symbol for the square root is  $\sqrt{\quad}$ . Calculating the square root of a number is the opposite of squaring the number.

For example, if  $5^2 = 25$  and  $-5^2 = 25$ , then  $\sqrt{25} = 5$  or  $-5$ .



### WORKED EXAMPLE 5 Calculating powers and square roots of integers.

Calculate each of the following.

a.  $(-3)^3$

b. The square root of 64

**THINK**


- a. 1. Write the expression in expanded form.
2. Evaluate by working from left to right, remembering that the answer will be negative because the power is odd.
- b. Look for the numbers that, when squared, result in 64 ( $8 \times 8 = 64$  and  $-8 \times -8 = 64$ ).

**WRITE**

a.  $(-3)^3 = (-3) \times (-3) \times (-3)$   
 $= +9 \times (-3)$   
 $= -27$

b.  $\pm\sqrt{64} = +8$  or  $-8$   
 $= \pm 8$

### on Resources

-  **Digital documents** SkillsHEET Multiplying integers (doc-6390)  
WorkSHEET Integers I (doc-6400)



## 1.3 Exercise

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1. **WE4** Evaluate each of the following.

a.  $-3 \times 5$

b.  $3 \times -7$

c.  $-6 \times -5$

d.  $2 \times -10$

e.  $-8 \times -5$

f.  $-7 \times 8$

2. **WE4** Evaluate each of the following.

a.  $-10 \times 25$

b.  $-125 \times 10$

c.  $-6 \times 9$

d.  $+9 \times -7$

e.  $-11 \times -7$

f.  $250 \times -2$

3. Use an appropriate method to evaluate the following.

a.  $-2 \times 3 \times -5 \times -10$

b.  $6 \times -1 \times 5 \times -2 \times 1$

c.  $8 \times -3 \times -1 \times -2 \times 4$

d.  $-3 \times 5 \times -2 \times -1 \times -1 \times -1$

e.  $-5 \times 6 \times -2 \times -2$

f.  $6 \times -1 \times 3 \times -2$

4. Complete the following equations.

a.  $6 \times \square = -42$

b.  $-5 \times \square = -30$

c.  $15 \times \square = -30$

d.  $\square \times -4 = 24$

e.  $\square \times -9 = 36$

f.  $\square \times 11 = -77$

5. **WE5aWE5b** Complete the following equations.

a.  $\square \times -12 = 108$

b.  $\square \times -11 = -66$

c.  $\square \times 4 = -48$

d.  $-10 \times \square = 80$

e.  $-9 \times \square = -81$

f.  $-12 \times \square = 144$

6. **WE5a** Evaluate each of the following.

a.  $(-2)^3$

b.  $(-3)^2$

c.  $(-2)^4$

d.  $(-3)^4$

e.  $(-2)^5$

f.  $(-9)^2$

7. **WE5a** Evaluate each of the following.

a.  $(-4)^2$

b.  $(-5)^3$

c.  $(-4)^4$

d.  $(-5)^4$

e.  $(-6)^3$

f.  $(-8)^3$

8. Complete the following statements.

a. If a negative number is raised to an even power, the answer is (positive/negative).

b. If a negative number is raised to an odd power, the answer is (positive/negative).

9. **WE5b** Evaluate the square root of the following numbers.

a. 121

b. 100

c. 36

d. 16

e. 9

f. 144

10. For each of the following, write three possible sets of integers that can be placed in the boxes to make the equation a true statement.

a.  $\square \times \square \times \square = -16$

b.  $\square \times \square \times \square = 24$

11. For each of the following, determine whether the result is a positive or negative value. You do not have to work out the value.

a.  $-47 \times -52 \times 100$

b.  $-56 \times -8 \times -66$

c.  $21 \times -21 \times 42 \times -32$

d.  $40 \times -5 \times 10 \times -2$

12. Explain what happens when a number is multiplied by  $-1$ , using examples to justify your answer.
13. Explain why the answer to this question is negative.

$$-2 \times -4 \times +3 \times -6 \times +4 \times +3$$

14. Evaluate  $(-1)^n \times (+1)^{n+1}$  if:

a.  $n$  is even

b.  $n$  is odd.

15. In a Year 11 Mathematics examination, there are 30 multiple choice questions. Students are given 2 marks for a correct answer,  $-1$  mark for an incorrect answer and zero marks for an unanswered question. A student scored a total of 33 marks in the multiple choice section. Explain how they could have reached this total.

## 1.4 Dividing integers

### LEARNING INTENTION

At the end of this subtopic you should be able to:

- divide integers
- use numerals, symbols, number facts and operations and strategies for calculation
- solve practical problems which require the use and application of a range of numerical computations involving integers, decimals, fractions, proportions, percentages, rates, powers and roots.



### 1.4.1 Division of integers

eles-xxxx

Division is the inverse or opposite operation of multiplication. We can use the multiplication facts for integers to discover the division facts for integers.

Multiplication fact	Division fact	Multiplication pattern	Division pattern
$4 \times 5 = 20$	$20 \div 5 = 4$ and $20 \div 4 = 5$	positive $\times$ positive = positive	$\frac{\text{positive}}{\text{positive}} = \text{positive}$
$-4 \times -5 = 20$	$20 \div -5 = -4$ and $20 \div -4 = -5$	negative $\times$ negative = positive	$\frac{\text{positive}}{\text{negative}} = \text{negative}$
$-4 \times 5 = -20$	$20 \div 5 = -4$ and $-20 \div -4 = 5$	negative $\times$ positive = negative and positive $\times$ negative = negative	$\frac{\text{negative}}{\text{positive}} = \text{negative}$ and $\frac{\text{negative}}{\text{negative}} = \text{positive}$

### Determining the sign of the answer when dividing integers

- When dividing two integers with the same sign, the answer is positive.

$$+ \div + = +$$

$$- \div - = +$$



- When dividing two integers with different signs, the answer is negative.

$$\begin{aligned} + \div - &= - \\ - \div + &= - \end{aligned}$$

- Remember that division statements can be written as fractions and then simplified.  
For example:

$$\begin{aligned} -12 \div -4 &= \frac{-12}{-4} \\ &= \frac{12 \times \cancel{1}}{4 \times \cancel{1}} \\ &= 3 \end{aligned}$$

### WORKED EXAMPLE 6 Dividing integers

Evaluate each of the following.

a.  $-48 \div 6$

b.  $\frac{-54}{-9}$

c.  $144 \div -6$

d.  $-240 \div -16$

**THINK**

a. The two numbers have different signs, so the answer is negative.

**WRITE**

a.  $-48 \div 6$   
 $= -8$

b. The two numbers have the same sign, so the answer is positive.

b.  $\frac{-54}{-9} = \frac{-1 \times 54}{-1 \times 9}$   
 $= \frac{54}{9}$   
 $= 6$

c. 1. Complete the division as if both numbers were positive numbers.

a.  $\begin{array}{r} 24 \\ 6 \overline{)144} \end{array}$

2. Determine the sign of the answer. The two numbers have different signs, so the answer is negative.

$144 \div -6 = -24$

d. 1. Complete the division as if both numbers were positive numbers.

b.  $\begin{array}{r} 15 \\ 16 \overline{)240} \end{array}$

2. Determine the sign of the answer. The two numbers have the same sign, so the answer is positive.

$-240 \div -16 = 15$

**on** Resources

 **Digital document** SkillsHEET Dividing integers (doc-6391)

## 1.4 Exercise

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1. **WE6a** Evaluate each of the following.

a.  $-54 \div 9$

b.  $10 \div -2$

c.  $-8 \div -2$

d.  $-5 \div -1$

e.  $99 \div -11$

f.  $0 \div -9$

2. Evaluate each of the following.

a.  $42 \div -3$

b.  $-130 \div 5$

c.  $-56 \div -8$

d.  $+88 \div -4$

e.  $-66 \div -6$

f.  $168 \div -8$

3. **WE6b** Evaluate each of the following.

a.  $\frac{-132}{-12}$

b.  $\frac{-16}{4}$

c.  $\frac{-40}{-8}$

d.  $\frac{28}{-7}$

e.  $\frac{-250}{-50}$

f.  $\frac{144}{-12}$

4. **WE7** Evaluate the following.

a.  $184 \div -8$

b.  $-189 \div 9$

c.  $-161 \div -7$

d.  $-132 \div -2$

e.  $-204 \div 6$

f.  $1080 \div -9$

5. **WE7** Evaluate the following.

a.  $216 \div -12$

b.  $-345 \div 15$

c.  $-1536 \div -24$

d.  $-1764 \div -49$

e.  $4095 \div 64$

f.  $-2695 \div 55$

6. Write three different division statements, each of which has an answer of  $-6$ .

7. Copy and complete the following by placing the correct integer in the box.

a.  $-36 \div \square = -9$

b.  $-72 \div \square = -9$

c.  $72 \div \square = -36$

d.  $-24 \div \square = 4$

e.  $\square \div 6 = -5$

f.  $\square \div 4 = -12$

8. Calculate the value of each of the following by working from left to right.

a.  $-48 \div 6 \div -4$

b.  $-240 \div 12 \div -5$

c.  $400 \div -5 \div 8 \div -2$

9. Copy and complete these tables.

a.

$\times$			$-6$	$+8$
			18	
$-10$		$-40$		
	10		30	
$-7$				$-56$

b.

$\times$			$-9$	
$6$	30			$-42$
		36		
	$-55$		99	
		$-6$	$-18$	

10. Copy and complete these tables. Divide the number on the top by the number on the side.

a.

÷	4	-10	12	-8
-2				
7				
-3				
-10				

b.

÷				-4
		-2		
-8	-4	3		
6			-6	
				1

11. Given  $\frac{(-1)^n}{(-1)^m}$ , state the values of  $m$  and  $n$  that would make this fraction positive.

12. Ross played in a recent golf tournament. He scored  $-3, +1, -4, +2$  compared to par. Calculate his average score compared to par.

(Hint: To calculate the average score, divide the sum of the scores by the number of scores.)

## 1.5 Combining operations on integers

### LEARNING INTENTION

At the end of this subtopic you should be able to:

- apply the order of operations to solve an equation
- check the reasonableness of answers.

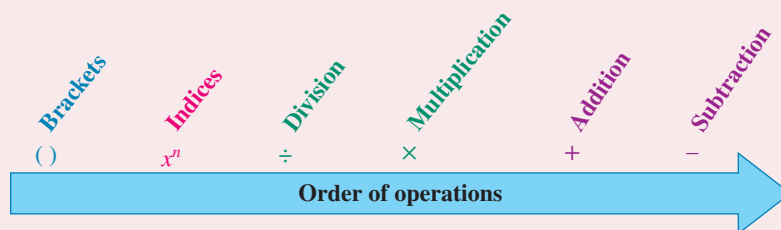
### 1.5.1 Order of operations

eles-xxxx

There are rules that determine which parts of an equation you need to calculate first, when the equation contains multiple operations. The term BIDMAS is used to remember the correct order in which to complete operations within an equation.

#### The order of operations

BIDMAS helps us to remember the correct order in which we should perform the various operations, working from left to right.





Order	Operations	What does it mean?
First	Brackets	Calculate any parts of the expression that are shown in brackets. For example: $(3 + 1) \times 4 = 4 \times 4$
Second	Indices	Multiply or divide out any indices. Indices are: <ul style="list-style-type: none"> <li>powers (<math>3^2</math>), where a number is multiplied by itself  <math>3^2 = 3 \times 3</math>  <math>= 9</math>  <math>3^3 = 3 \times 3 \times 3</math>  <math>= 27</math></li> <li>roots (<math>\sqrt{9}</math>), the opposite of a power.  <math>\sqrt{9} = 9 \div 3</math>  <math>= 3</math>  <math>\sqrt[3]{27} = 27 \div 3 = 9</math></li> </ul>
Third	Division and Multiplication	Calculate any parts of the expression that involve division or multiplication. If the expression contains both multiplication and division, start from the left and work across to the right. For example: $3 \times 4 + 36 \div 6 = 12 + 6$ $= 18$
Last	Addition and Subtraction	Calculate any parts of the expression that involve addition or subtraction. If the expression contains both addition and subtraction, start from the left and work across to the right. For example (remember division comes before addition or subtraction): $4 + 24 \div 4 - 2 = 4 + 6 - 2$ $= 10 - 2$ $= 8$

### WORKED EXAMPLE 7 Applying the order of operations

Calculate the value of each of the following.

a.  $23 - 6 \times 4$

b.  $(12 - 8) + 5^2 - (10 + 2^2)$

c.  $\frac{3(4 + 8) + 4}{4 + 2(3^2 - 1)}$

#### THINK

a. 1. Apply BIDMAS to determine the first step (in this instance multiplication).

2. Complete the next step in the calculation (in this instance subtraction) and write the answer.

b. 1. Apply BIDMAS to determine the first step (perform the calculations in brackets first, then remove the brackets).

2. Complete the next steps in the calculation (resolve the powers, then carry out addition and subtraction from left to right).

#### WRITE

a.  $23 - 6 \times 4$   
 $= 23 - 24$   
 $= -1$

b.  $(12 - 8) + 5^2 - (10 + 2^2)$   
 $= 4 + 5^2 - (10 + 4)$   
 $= 4 + 5^2 - 14$   
 $= 4 + 25 - 14$   
 $= 29 - 14$

3. Complete the final step and write the answer.
- c. 1. Apply BIDMAS to determine the first step (brackets).

2. Complete the next step in the calculation (multiplication).

3. Perform the calculations on the numerator and denominator separately.

4. Complete the division and write the answer.

$$= 15$$

$$c. \frac{3(4+8)+4}{4+2(3^2-1)} = \frac{3 \times 12 + 4}{4 + 2(9-1)}$$

$$= \frac{3 \times 12 + 4}{4 + 2 \times 8}$$

$$= \frac{36 + 4}{4 + 16}$$

$$= \frac{40}{20}$$

$$= 2$$

## 1.5.2 Reasonableness

eles-xxxx

When calculating mathematical answers, it is always important to understand the question so you have an idea of what a reasonable answer would be. Checking the reasonableness of answers can indicate possible mistakes in your working.

### WORKED EXAMPLE 8 Checking for reasonableness of an answer

Cathy goes shopping to purchase an outfit for a hike. She buys a \$145 pair of waterproof pants and a \$180 jacket, and gets \$30 off the combined price by purchasing the two together. She also buys a pair of boots for \$120 and a pair of socks for \$15. Calculate how much Cathy spent in total and check your answer for reasonableness.



#### THINK

1. Read the question carefully to understand what it is about.
2. Have an idea of what sort of answer you expect.
3. Write a mathematical expression to calculate the total amount that Cathy spent, including the discount.
4. Check the answer for reasonableness.

#### WRITE

The question asks you to add up Cathy's total shopping bill including the discount.

Cathy purchased three items, each over \$100 dollars, so the answer should be over \$300.

$$\begin{aligned} \text{Cathy spent} &= \$145 + \$180 - \$30 + \$120 + \$15 \\ &= \$430 \end{aligned}$$

This answer is above \$300, so it seems reasonable.

## Resources

- Digital documents** SkillSHEET Order of operations (doc-6415)  
SpreadSHEET Four operations (doc-2090)
- Video eLesson** BIDMAS (eles-2435)
- Interactivity** Order of operations (int-3707)

## 1.5 Exercise

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1. **WE8** Calculate the values of the following.

a.  $-4 \times 2 + 1$   
c.  $9 \times (8 - 3)$   
e.  $4 + 12 \times -5$

b.  $8 \div (2 - 4) + 4$   
d.  $-3 - 40 \div 8 + 2$   
f.  $-5 \times 12 + 2$

2. Calculate the values of the following.

a.  $12 - 6 \div 3$   
c.  $(17 - +7) \div -5$   
e.  $100 \div (-50 \div -2) + 10$

b.  $45 \div (27 \div -3)$   
d.  $-12 + 8 \times 7$   
f.  $9 + \frac{24}{-6} \times 3$

3. Calculate the values of the following.

a.  $-7 + 4 \times -4$   
c.  $(-5)^2 - 3 \times -5$   
e.  $52 \div (-9 - 4) - 8$

b.  $(-63 \div -7) \times -3 + -2$   
d.  $-6 \times -2 - [3 + (-6)^2]$   
f.  $-6 - 64 \div -16 + 8$

4. **WE9** Kyle went to the sports department to get some clothes for the gym. He purchased a \$120 tracksuit and a pair of \$150 runners as a package, and got \$35 off the combined price.

Kyle also purchased a pair of running shorts for \$30 and a singlet top for \$25.

Determine how much Kyle spent at the sports department. Check your answer for reasonableness.



5. Bob goes shopping for some party food. He buys 6 packets of chips at \$2.50 each, and 2 boxes of soft drink at \$8.50 each. Since he purchases 2 boxes of soft drink, he gets a \$2 discount. He also buys 5 packets of biscuits at \$1 each and 3 dips at \$2 each.

Determine how much Bob spends in total and check your answer for reasonableness.

6. Erica went to the warehouse to get some building supplies. She bought three 4-m-long pieces of timber at \$4.50 per metre, two packets of nails at \$5.50 each and three tubes of liquid nails at \$4 each.

Because Erica is a regular customer, she received a \$15 discount off the total purchase price.

Determine how much Erica spent in total and check your answer for reasonableness.





7. Taiki headed north on a bike ride, initially travelling 25 km. He then turned around and travelled 15 km south before stopping for a drink. After his drink, Taiki continued to ride south for another 20 km, before again turning around and travelling north for a further 25 km.
- Calculate how many kilometres Taiki covered on his ride.
  - Determine how far north he finished from where he started.
8. Students were given the following question to evaluate.



$$4 + 8 \div (-2)^2 - 7 \times 2$$

- A number of different answers were obtained, including  $-8$ ,  $-12$  and  $-17$ . Determine which one of these is correct.
  - Using only brackets, change the question in two ways so that the other answers would be correct.
9. Calculate the number required to make the following equation true:

$$13 \times (15 - 1) = 180 + \square$$

10. Calculate the number required to make the following equation true:

$$(10^2 + 12 \div 3) \div (-8) = \square$$

# 1.6 Review

## 1.6.1 Summary

doc-12345

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## 1.6 Exercise

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### Multiple choice

- MC** State which of the following is an integer.

A. $-1.3$	B. $\frac{3}{4}$	C. $23$
D. $0.25$	E. $1.234$	
- MC** State which of the following is incorrect.

A. $5 > 1$	B. $-5 < -1$	C. $-25 < 0$
D. $-25 > 1$	E. $25 < 1$	
- MC** The integers between  $-11$  and  $-7$  are:

A. $-10, -9, -8$	B. $10, 9, 8$	C. $-12, -13, -14$
D. $-10.5, -9.5, -8.5$	E. $-8\frac{1}{2}, -9\frac{1}{2}, -11\frac{1}{2}$	
- MC** Arrange the following from lowest to highest:  $-12, 34, 0, -3, 7$

A. $0, -3, 7, -12, 34$	B. $-12, 34, 0, -3, 7$	C. $-3, 7, -12, 34, 0$
D. $34, 7, 0, -3, -12$	E. $-12, -3, 0, 7, 34$	
- MC** The value of the expression  $17 + 3 \times 7$  is:

A. $27$	B. $140$	C. $38$
D. $28$	E. $357$	
- MC** The value of the expression  $-3(5 - -10) + 50$  is:

A. $95$	B. $5$	C. $65$
D. $-5$	E. $10$	
- MC** State which of the following statements is true.

  - Multiplying an even number of negative numbers together gives a negative answer.
  - The only square root of  $25$  is  $+5$ .
  - Subtracting a positive number from another positive number always gives a negative number.
  - Adding two negative numbers together gives a positive answer.
  - Dividing a negative number by another negative number gives a positive answer.

8. **MC** Determine which of the following has an answer of  $-4$ .

A.  $2 + 2$

B.  $2 - +2$

C.  $-2 + 2$

D.  $-2 + -2$

E.  $2^2$

9. **MC** Determine which of the following has an answer of  $-3$ .

A.  $\frac{-24}{-8}$

B.  $\frac{-8}{24}$

C.  $\frac{27}{-9}$

D.  $\frac{-27}{-9}$

E.  $\frac{-30}{-10}$

10. **MC** Determine which of the following has an answer of  $+36$ .

A.  $-3 \times -12$

B.  $-9 \times +4$

C.  $-3 \times -6$

D.  $+6 \times -6$

E.  $-3 \times -11$

### Short answer

11. Calculate the following.

a.  $-7 + -13$

b.  $23 - -18$

c.  $-4 + -5 - +3$

d.  $-2 - -7 + -5$

12. Calculate the following.

a.  $-4 \times -9$

b.  $-7 \times +8$

c.  $\sqrt{81}$

d.  $-(-3)^3$

13. Calculate the following.

a.  $-24 \div +6$

b.  $72 \div -12$

c.  $\frac{+56}{-7}$

d.  $\frac{-144}{-12}$

14. Calculate the following.

a.  $36 \div 9 \times 8 - 12$

b.  $45 - 15 \div 3$

c.  $30 - 10 \times 2 + 7$

d.  $64 - (16 + 8) + 26$

15. Fill in the missing number.

a.  $13 - \square = 18$

b.  $-23 + \square = 7$

c.  $\square - 4 - -5 = 3$

d.  $-32 - \square = -18$

### Extended response

16. Scott bought some pipe to complete the downpipes on his new carport. He purchased 34.2-m lengths at \$5.15 per metre and 23.6-m lengths at \$6.75 per metre. He also purchased some 5 fittings at \$1.25 each. Calculate how much Scott spent on the downpipes.

17. Yi Rong is saving for her end-of-year holiday with her school friends. Her parents said if she did work around the house they would help contribute to the holiday expenses. They agreed to pay her \$25 a week for doing the dishes and \$15 a week for doing the washing; for extra incentive, they said they would double her pay if she did each of the jobs for the next 4 weeks without missing a day.

Assuming Yi Rong didn't miss a day, calculate how much money she earned in the 4 weeks.



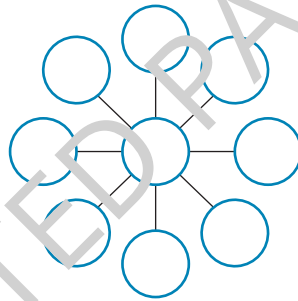
18. On a test, each correct answer scores 5 points, each incorrect answer scores  $-2$  points and each unanswered question scores 0 points. Calculate the following students' test scores.

- a. Student 1: 17 correct answers, 2 incorrect answers and 1 question unanswered
- b. Student 2: 13 correct answers, 5 incorrect answers and 2 questions unanswered

19. Answer the following questions.

- a. You have \$75 and you spend \$15 on soft drink and another \$8 on chips. A friend gives you \$6 to buy lunch, which cost \$16 for the two of you. You decide to also buy an ice cream for \$3. Calculate how much money you have left.
- b. Two friends saved \$895 together for their holiday. They spend \$350 on accommodation and then \$125 each to go whale watching. Due to bad weather, the trip was cut short, so they got back \$50 each. They then decided to go out to dinner and together spent \$148. After the holiday, they split the remaining money in half. Calculate how much each friend got back.

20. Insert the integers from  $-6$  to  $+2$  into the circles on the this figure below so that each line of three circles has each of the following totals.



a.  $-6$

b.  $-3$

c.  $-9$



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# Answers

## Topic 1 Integers

### 1.2 Adding and subtracting integers

#### 1.2 Exercise

- 5, -2, 212, -33
- 3
  - 5
  - 2
  - 11
  - 30
  - 13
- 4
  - 1
  - 1
  - 0
  - 5
  - 2
- 10
  - 15
  - 8
  - 1
  - 39
  - 46
- 5
  - 10
  - 6
  - 9
  - 11
  - 37
- 49
  - 35
  - 32
  - 12
  - 25
  - 172
- 14
  - 31
  - 18
  - 9
  - 25
  - 0
- 7
  - 4
  - 12
  - 5
  - 4
  - 5
- True
  - False
  - False
  - True
  - False
  - False
- 4 and -4
  - 2 and -2
  - 10 and -10
- 7 and -7
  - 11 and -11
  - 13 and -13
- 60 m to the right of where she started
- 39°C
  - 159°C
  - 198°C
- 7°C

### 1.3 Multiplying integers

#### 1.3 Exercise

- 15
  - 21
  - 30
  - 20
  - 40
  - 36
- 250
  - 1250
  - 500
  - 63
  - 77
  - 500
- 300
  - 60
  - 192
  - 30
  - 120
  - 36
- 7
  - 6
  - 2
  - 6
  - 4
  - 7
- 9
  - 6
  - 12
  - 8
  - 9
  - 12
- 8
  - 9
  - 16
  - 81
  - 32
  - 81
- 16
  - 125
  - 256
  - 625
  - 216
  - 512
- Positive
  - Negative
- $\pm 11$
  - $\pm 10$
  - $\pm 6$
  - $\pm 4$
  - $\pm 3$
  - $\pm 12$
- $2 \times 2 \times -4 = -16$
  - $-4 \times 2 \times 2 = -16$
  - $2 \times -4 \times 2 = -16$

There are more options.

- $3 \times 2 \times 4 = 24$   
 $3 \times -2 \times -4 = 24$   
 $-3 \times -2 \times 4 = 24$   
 There are more options.

- Positive
  - Negative
  - Positive
  - Positive
- When multiplying by 1, the number stays the same. So when multiplying by -1, the number becomes negative.
- The answer will be negative, since there are three negative numbers (odd number).
- Positive
  - Negative
- One option could be 20 correct answers, 7 incorrect answers and 3 unanswered.  
 $(20 \times 2) + (7 \times -1) + (3 \times 0) = 40 - 7 + 0$   
 $= 40 - 7$   
 $= 33$

### 1.4 Dividing integers

#### 1.4 Exercise

- 6
  - 5
  - 4
  - 5
  - 9
  - 0
- 14
  - 26
  - 7
  - 22
  - 11
  - 21
- 1
  - 4
  - 5
  - 4
  - 5
  - 12
- 23
  - 21
  - 23
  - 66
  - 34
  - 120
- 18
  - 23
  - 64
  - 36
  - 64
  - 49
- There are many other answers.  
 $36 \div -6 = -6$   
 $-54 \div 9 = -6$   
 $72 \div -12 = -6$
- 4
  - 8
  - 2
  - 6
  - 30
  - 48
- 2
  - 4
  - 5

9. a.

$\times$	-2	4	-6	+8
3	-6	12	18	24
-10	20	-40	60	-80
-5	10	-20	30	-40
-7	14	-28	42	-56

b.

$\times$	5	-3	-9	-7
6	30	-18	-54	-42
-12	-60	36	108	84
-11	-55	33	99	77
2	10	-6	-18	-14

10. a.

÷	4	-10	12	-8
-2	-2	5	-6	4
7	$\frac{4}{7}$	$-\frac{10}{7}$	$\frac{12}{7}$	$-\frac{8}{7}$
-3	$-\frac{4}{3}$	$\frac{10}{3}$	-4	$\frac{8}{3}$
-10	$-\frac{2}{5}$	1	$-\frac{6}{5}$	$\frac{4}{5}$

b.

÷	32	-24	-36	-4
12	$\frac{8}{3}$	-2	-3	$-\frac{1}{3}$
-8	-4	3	$\frac{9}{2}$	$\frac{1}{2}$
6	$\frac{16}{3}$	-4	-6	$-\frac{2}{3}$
-4	-8	6	9	1

11. For the numerator (top) and the denominator (bottom), positive  $n$  and  $m$  need to be even numbers.  
For the numerator (top) and the denominator (bottom), negative  $n$  and  $m$  need to be odd numbers.

12. -1

## 1.5 Combining operations on integers

### 1.5 Exercise

1. a. -7                      b. 0                      c. 45  
d. -6                      e. -56                  f. -58
2. a. 10                      b. -5                      c. -2  
d. 44                      e. 14                      f. -3
3. a. -23                    b. -29                    c. 40  
d. 9                        e. -12                    f. 5

4. \$290

5. \$41

The answer is reasonable, since chips  $\approx$  \$10, soft drink  $\approx$  \$20 biscuits  $\approx$  \$5 and chips  $\approx$  \$5.

This totals \$40, which reduces to \$38 after the discount. This value is close to \$41, so it is reasonable.

6. \$62

The answer is reasonable since timber  $\approx$  \$60, nails  $\approx$  \$10 and liquid nails  $\approx$  \$1.

This totals \$71, which reduces to \$25 after the discount. This value is close to \$26, so it is reasonable.

7. a. 85 km                    b. 15 km north

8. a. -8

b.  $4 + 8 \div -(2)^2 - 7 \times 2 = -12$

$(4 + 8) \div -(2)^2 - 7 \times 2 = -17$

9. 2

10. -13

## 1.6 Review

### 1.6 Exercise

1. C  
2. D

3. A

4. E

5. C

6. B

7. E

8. D

9. C

10. A

11. a. -20

b. 41

c. -15

d. 0

12. a. 36

b. -56

c.  $\pm 9$

d. 27

13. a. -4

b. -6

c. -8

d. 12

14. a. 20

b. 40

c. 17

d. 66

15. a. -5

b. 30

c. 2

d. -14

16. \$341.68

17. \$320

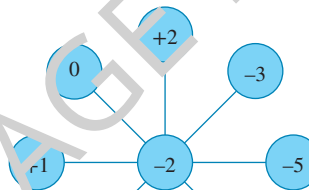
18. a. 81

b. 15

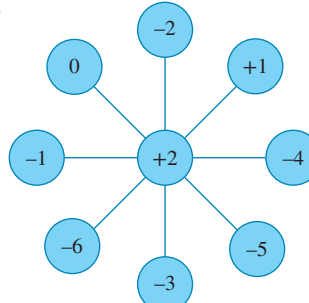
19. a. \$39

b. \$1,350

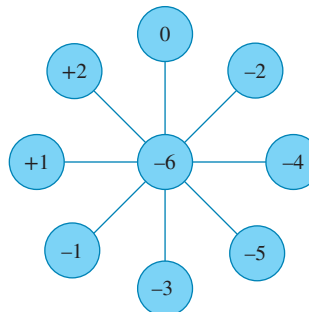
20. a.



b.



c.



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